Scalar mesons nonet in a scaled effective Lagrangian

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Abstract

A scaled SU(3) Nambu - Jona-Lasinio Lagrangian is used to compute the mass of the nine scalar mesons in the vacuum and the mass of the gluball. It is shown that a suitable choice of the vacuum gluon condensate allows to reproduce the experimental masses of the scalar mesons except for the $K_0^*(1430)$. This choice corresponds to a weak coupling between the gluon and quark condensates, giving a $f_0(1500)$ or a $f_J(1710)$ which is nearly a pure glueball.

Key words: NJL, Scalar mesons, Glueball
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1 Introduction

Solving the scalar mesons puzzle is one of the most important problems in meson spectroscopy. The identification of the scalar nonet is all but clear and literature is full of contradictory statements. The problem is still complicated by the presence of glueballs. Following Montanet [1], an attractive scenario would be to include the $a_0(1450)$, $K_0^*(1430)$, $f_0(1300)$ and $f_0(1525)$ or $f_0(1590)$ in the nonet. This choice would leave out the $a_0(980)$ and the $f_0(980)$ which could be identified with $K\bar{K}$ molecule states. The glueball candidate could be speculated to be the $f_0(1500)$. Palano [2] offers another scenario according to which the nonet would be the $a_0(980)$, $K_0^*(1430)$,$f_0(980)$ and the $f_0(1400)$. The scalar glueball could be identified with the $f_0(1520)$. Surprisingly, its low mass meson $f_0(980)$ seems to be a $s\bar{s}$ state while the high mass one $f_0(1400)$ would be a $u\bar{u} + d\bar{d}$ state. In the same idea, Törnqvist [3] presents the two mesons $f_0(980)$ and $f_0(1300)$ as two manifestations of the same $s\bar{s}$ state. The chiral partner of the pion (the $u\bar{u} + d\bar{d}$ state) should be identified with a very broad resonance (Γ = 880 MeV) centered at 860 MeV. According to Lindenbaum and Longacre [4], the 0^{++} data can be reproduced with the four

mesons $f_0(980)$, $f_0(1300)$, $f_0(1400)$ and $f_0(1710)$. They then suppose that the $\theta(1710)$ (here $f_0(1710)$) has spin 0 and not spin 2 that is far but evident [2,5,6]. These exemples show that the situation is quite confused from an experimental point of view.

Things are not much better from the theoretical one. Klempt *et al.* [7] calculate the masses and some of the decay properties of the scalar nonet, using a relastivistic quark model with linear confinement and an instanton-induced interaction. Their results suggest to identify the nonet with $a_0(1450)$, $K_0^*(1430)$, $f_0(980)$ and $f_0(1500)$, the later being a $s\bar{s}$ state. No glueball is included in their calculations. Lattice QCD calculations provide an idea of the value of the glueball mass. However, there is still some dissention on this point. Initially, the predictions for the mass of the lightest gluball seemed to favour its identification with the $f_0(1500)$ [8,9]. Recently, the valence (quenched) approximation developped by Sexton *et al.* [10] has predicted the glueball as being the $f_J(1710)$. Moreover, this $f_J(1710)$ should be nearly a pure gluball, the mixing with $q\bar{q}$ states increasing its mass by around 60 MeV. The fact that the pure glueball state disperses over three resonances is also confirmed by the recent analysis of Anisovitch *et al.* [11]. However, this analysis seems to indicate that the mixing with $q\bar{q}$ decreases the mass of the glueball.

The scaled Nambu - Jona-Lasinio model introduced in Ref. [12] in the SU(2) case and its extended version to the SU(3) case [13] allows for such a mixing between the quark and the glue sectors. In the present paper, we compute the masses of the scalar nonet and of the gluball. Our model contains two free parameters. One of them, the constituent up quark mass, is choosen to reproduce the $a_0(980)$. We show that for a relatively large value of the vacuum gluon condensate ($\chi_0 = 350$ MeV), the scalar nonet assumed to be $a_0(980)$, $K_0^*(1430)$, $f_0(980)$ and $f_0(1370)$ [14] can be well reproduced (except the $K_0^*(1430)$). This large value of χ_0 amounts to a weak coupling between the quark and the gluon condensates. The quark contents of the gluball increases its mass by less than 20 MeV.

The paper is organized as follow. Sec. 2 recalls the usefull tools of the scaled NJL model. Sec. 3 specifies our presciption used to avoid the threshold problem when the meson masses lie above the quark-antiquark pair creation threshold. Our results concerning the masses of the mesons and of the glueball are given in Sec. 4. Finally, Sec. 5 draws our conclusions.

2 The model

The model used in this paper is extensively described in Ref. [13,15] under the name of "A-scaling model". We just recall some of the usefull tools for the

understanding of the present work. We start from the vacuum SU(3) effective Euclidean action:

$$I_{eff}(\varphi,\chi) = -\mathrm{Tr}_{\Lambda\chi}(-i\partial_\mu\gamma_\mu + m + \Gamma_a\varphi_a) + \int d^4x \frac{a^2\chi^2}{2}\varphi_a\varphi_a + \int d^4x \mathrm{L}_\chi (1)$$

which is written in its bosonized version for which the quark degrees of freedom have been integrated out. The meson fields write:

$$\varphi_a = (\sigma_a, \pi_a), \qquad \Gamma_a = (\lambda_a, i\gamma_5\lambda_a), \qquad a = 0, ..., 8 \tag{2}$$

where the λ_a are the usual Gell-mann matrices with $\lambda_0 = \sqrt{2/3}\mathbf{1}$. We choose to work in the isospin symmetry limit and the quantity m stands for the diagonal matrix $diag(m_u, m_u, m_s)$. The trace anomaly of QCD is modelized using a scalar dilaton field χ that is intimately related to the gluon condensate $\chi \propto \langle G^2_{\mu\nu}\rangle^{1/2}$ [16]:

$$\mathrm{L}_\chi = \frac{1}{2}(\partial_\mu\chi)^2 + \frac{1}{16}\mathrm{b}^2(\chi^4 \ln\frac{\chi^4}{\chi^4_{\mathrm{G}}} - (\chi^4 - \chi^4_{\mathrm{G}})). \tag{3}$$

Since we are only interested in the scalar sector, the axial anomaly which would give the η-η' mass difference is not considered here. The model contains six parameters: the current quark masses (m_u, m_s), the strengths (a^2, b^2), the gluon parameter χ_G and the cut-off Λ introduced to regularised the diverging quark loop. Four of these parameters are adjusted to reproduce the pion mass (m_π), the weak pion decay constant (f_π), the kaon mass (m_K) and the mass of the glueball (m_{GL}) which is sometimes identified with the $f_0(1500)$ and othertimes with the $f_J(1710)$. We will perform calculations with these two different values. We are then left with two free parameters that we choose to be the constituent up quark mass M^0_u (related to a^2) and the vacuum gluon condensate χ_0 (related to χ_G). These free vacuum parameters as well as the vacuum constituent strange quark mass M^0_s correspond to the stationnary point of the effective action. They then satisfy three coupled equations that can be found in Ref. [13]. We do not repeat them here.

The mass of the various mesons are obtained by expanding the effective action up to second order in the fluctuating parts of the meson fields $(\tilde{\sigma_a}, \tilde{\pi_a})$ and of the dilaton field $\tilde{\chi}$:

$$I^{(2)}(\varphi,\chi) = \frac{1}{2\beta\Omega}\sum_q \left(\tilde{\phi}_{aq}\Sigma^{-1}_{ab}\tilde{\phi}_{b-q} + \tilde{\pi}_{aq}\Pi^{-1}_{ab}\tilde{\pi}_{b-q}\right), \tag{4}$$

where $\tilde{\phi}_a = (\tilde{\sigma}_a, \tilde{\chi})$. The matrix Π^{-1} is a 9 X 9 matrix while the corresponding scalar matrix Σ^{-1} we are interested in is 10 X 10. It only mixes the three fields

$(\tilde{\sigma}_0, \tilde{\sigma}_8)$ and $\tilde{\chi}$ so that one can write:

$$\frac{1}{2\beta\Omega}\sum_q \tilde{\phi}_{aq}\Sigma^{-1}_{ab}\tilde{\phi}_{b-q} = \frac{1}{2\beta\Omega}\sum_q (\tilde{\sigma}_{0q}, \tilde{\sigma}_{8q}, \tilde{\chi}_q) S^{-1}\begin{pmatrix}\tilde{\sigma}_{0-q}\\ \tilde{\sigma}_{8-q}\\ \tilde{\chi}_{-q}\end{pmatrix}$$
$$+\frac{1}{2\beta\Omega}\sum_q\sum_{i=1}^{7}\left(\tilde{\sigma_{iq}} K^{-1}_{ii}\tilde{\sigma_{i-q}}\right). \tag{5}$$

The inverse propagators K^{-1}_{ii} take the simple forms:

$$K^{-1}_{ii} = 4N_c F(M^0_u, M^0_u)\left(q^2 + 4M^{0\,2}_u\right) + a^2\chi^2\frac{m_u}{M^0_u}, \qquad i = 1, 2, 3, \tag{6}$$

$$K^{-1}_{ii} = 4N_c F(M^0_u, M^0_s)\left(q^2 + \left(M^0_u + M^0_s\right)^2\right) + \frac{1}{2}a^2\chi^2\left(\frac{m_u}{M^0_u} + \frac{m_s}{M^0_s}\right),$$
$$i = 4, 5, 6, 7, \tag{7}$$

where

$$F(M^0_i, M^0_j) = \int d^4k \frac{1}{\left(k^2 + M^{0\,2}_i\right)\left((k-q)^2 + M^{0\,2}_j\right)}. \tag{8}$$

The zeroes of (6) and (7) give the mass of the $u\bar{u}$ and the $(u\bar{s}+s\bar{u})$ excitations, assumed to be the $a_0(980)$ and the $K^*_0(1430)$, respectively. In order to get the mass of the remaining scalar mesons and of the glueball, we will diagonalize the matrix S^{-1}

$$S^{-1} = \begin{pmatrix} S^{00} & S^{08} & S^{0\chi}\\ S^{80} & S^{88} & S^{8\chi}\\ S^{\chi 0} & S^{\chi 8} & S^{\chi\chi}\end{pmatrix} \tag{9}$$

and will search for the zeroes of its three eigenvalues. The corresponding physical meson fields will be denoted $\tilde{\Phi} = (\tilde{\Phi}_1, \tilde{\Phi}_2, \tilde{\Phi}_3)$. We shall see that $\tilde{\Phi}_1$ is compatible with $f_0(980)$, while $\tilde{\Phi}_2$ reproduces the $f_0(1300)$ [1] ($f_0(1400)$ [2], $f_0(1370)$ [14]). In the same way, $\tilde{\Phi}_3$ can be associated with the $f_0(1500)$ [1,2,9] or with the $f_J(1710)$ [10].

Before performing the total diagonalization, an intermediate step can help to get some physical insight in the quark contents of the various fields. It amounts

to diagonalize the matrix

$$\begin{pmatrix} S^{00} & S^{08} \\ S^{80} & S^{88} \end{pmatrix} \tag{10}$$

corresponding to the case of a NJL model without glue. One then has:

$$\frac{1}{2\beta\Omega}\sum_q(\tilde{\sigma}_{0q},\tilde{\sigma}_{8q},\tilde{\chi}_q)S^{-1}\begin{pmatrix} \tilde{\sigma}_{0-q} \\ \tilde{\sigma}_{8-q} \\ \tilde{\chi}_{-q} \end{pmatrix} = \frac{1}{2\beta\Omega}\sum_q\sum_{i,j=I,II,III}\tilde{\sigma}^T_{iq}(S_R^{-1})_{ij}\tilde{\sigma}_{j-q} \tag{11}$$

with

$$S_R^{-1} = \begin{pmatrix} S^{I,I} & 0 & S^{I,\chi} \\ 0 & S^{II,II} & S^{II,\chi} \\ S^{\chi,I} & S^{\chi,II} & S^{\chi,\chi} \end{pmatrix} \tag{12}$$

where the definition of $\tilde{\sigma}_I$ and $\tilde{\sigma}_{II}$ in terms of $\tilde{\sigma}_0$ and $\tilde{\sigma}_8$ can be found in Ref. [13] and where $\tilde{\sigma}_{III} = \tilde{\chi}$. It is helpfull to recall that

$$S^{I,I} = 4N_cF(M_u^0, M_u^0)\left(q^2 + 4M_u^{0\,2}\right) + a^2\chi^2\frac{m_u}{M_u^0} \tag{13}$$

and

$$S^{II,II} = 4N_cF(M_s^0, M_s^0)\left(q^2 + 4M_s^{0\,2}\right) + a^2\chi^2\frac{m_s}{M_s^0}. \tag{14}$$

These expressions show that $\tilde{\sigma}_I$ and $\tilde{\sigma}_{II}$ correspond to pure $u\bar{u}$ and $s\bar{s}$ excitations respectively . Due to the coupling with the dilaton field the physical fields $\tilde{\Phi}_I$ and $\tilde{\Phi}_{II}$ cease to be pure $u\bar{u}$ and $s\bar{s}$ excitations and $\tilde{\Phi}_{III}$ is not a pure glueball anymore. Our model then provides a way of giving some $q\bar{q}$ contents to the glueball. Depending on the value of the vacuum gluon condensate χ_0, the mixing between the fields can be large or not and so is this contents in $q\bar{q}$ of the glueball.

3 $q\bar{q}$ pair creation threshold

Due to the nonconfining disease of the NJL model, the various scalar mesons lie above the quark-antiquark pair creation threshold whatever is the choice for the free parameters $\overset{0}{M}_u$ and χ_0. Indeed, one can easily see from (6) and (7) that, due to the nonvanishing current quark masses, one always has:

$$m_{a_0} > 2\overset{0}{M}_u \qquad\qquad m_{K_0^*} > \overset{0}{M}_u + \overset{0}{M}_s. \tag{15}$$

Due to the mixing between the $\tilde{\sigma}_j$, $j = I, II, III$, the associated threshold is $2\overset{0}{M}_u$. Above these thresholds, the poles of the propagators become complex and the mesons acquire some width. For instance, the mass of the $a_0(980)$ and its width should verify:

$$4N_c F(-(m_{a_0} + i\epsilon - i\Gamma_{a_0})^2)\left(-(m_{a_0} - i\Gamma_{a_0})^2 + 4{M_u^0}^2\right) + a^2\chi^2\frac{m_u}{M_u^0} = 0 \tag{16}$$

where we have dropped the arguments $\overset{0}{M}_u$ in the fonction F (see Eq. (8)), for simplicity. Eq. (16) corresponds to two coupled equations which can not be decoupled without a suitable and model dependent prescription. In Refs. [17–19], the prescription amounted to replace Eq. (16) by:

$$4N_c F(-(m_{a_0} + i\epsilon)^2)\left(-(m_{a_0} - i\Gamma_{a_0})^2 + 4{M_u^0}^2\right) + a^2\chi^2\frac{m_u}{M_u^0} = 0. \tag{17}$$

Here, we perform calculations with the prescrition which amounts to replace the complex function F by its modulus, yielding a vanishing width and having the merit of simplicity. The mass of the $a_0(980)$ therefore satisfies:

$$4N_c|F(-(m_{a_0} + i\epsilon)^2)|\left(-{m_{a_0}}^2 + 4{M_u^0}^2\right) + a^2\chi^2\frac{m_u}{M_u^0} = 0. \tag{18}$$

Another prescription introduced recently [20] that could have been tempting to follow consists in introducing an infrared cut-off that eliminates all the divergent processes. If there were no coupling between quarks and glueball, this method should have been surely the simplest one. However, due to this mixing, the infrared cut-off has to be taken very large so that one should cut nearly all the momenta of the quark loop!

Table 1
Masses (MeV) of the scalar nonet for $m_{\Phi_{III}} = m_{f_0} = 1500$ MeV

	$\chi_0 = 350$ MeV	$\chi_0 = 200$ MeV	$\chi_0 = 125$ MeV
$a_0(980)$	980	980	980
$K_0^*(1430)$	1183	1183	1183
$m_{\Phi_I}[f_0(980)]$	973	949	798
$m_{\Phi_{II}}[f_0(1370)]$	1367	1331	1229
m_{σ_I}	980	980	980
$m_{\sigma_{II}}$	1383	1383	1383
$m_{\sigma_{III}}$	1482	1445	1355
Δ_I	$7.2\ 10^{-3}$	$3.3\ 10^{-2}$	$2.3\ 10^{-1}$
Δ_{II}	$1.2\ 10^{-2}$	$3.9\ 10^{-2}$	$1.3\ 10^{-1}$
Δ_{III}	$1.2\ 10^{-2}$	$3.7\ 10^{-2}$	$9.7\ 10^{-2}$
$\langle \bar{u}u \rangle^{\frac{1}{3}}$	-208	-208	-208
$\langle \bar{s}s \rangle^{\frac{1}{3}}$	-207	-207	-207

4 Results

We choose for the free parameter M_u^0 the relatively large value $M_u^0 = 489$ MeV that allows to reproduce the mass of the $a_0(980)$. Results for the masses of the scalar nonet are given in tables 1 and 2 for three specific values of the gluon condensate: $\chi_0 = 350$ MeV, 200 MeV and 125 MeV. The values of the quark condensates $\langle \bar{u}u \rangle$ and $\langle \bar{s}s \rangle$ are also indicated. The "experimental" values of the scalar mesons masses are taken from Ref. [14]. It is tempting to quantify the contents of the physical fields in the $u\bar{u}$, $s\bar{s}$, and in the glueball channels. One possible way is just to compare the exact masses with the approximated ones resulting from a vanishing coupling. The latter approximation defines the masses m_{σ_I}, $m_{\sigma_{II}}$ and $m_{\sigma_{III}}$. Tables 1 and 2 give the values of

$$\Delta_i = \frac{|m_{\Phi_i} - m_{\sigma_i}|}{m_{\Phi_i}} \qquad i = I, II, III. \tag{19}$$

When $m_{\Phi_{III}} = m_{f_0} = 1500$ MeV, the small values of Δ_i for $\chi_0 = 350$ MeV show that the physical mesons are nearly pure $u\bar{u}$, $s\bar{s}$ or glueball excitations while the mixing is large for $\chi_0 = 125$ MeV. For $\chi_0 = 200$ MeV, one finds that the nonet scalar is rather well reproduced except the mass of the $K_0^*(1430)$. According to [10], a mass difference of 60 MeV between the $f_0(1500)$ and the

Table 2
Masses (MeV) of the scalar nonet for $m_{\Phi_{III}} = m_{f_J} = 1710$ MeV

	$\chi_0 = 350$ MeV	$\chi_0 = 200$ MeV	$\chi_0 = 125$ MeV
$a_0(980)$	980	980	980
$K_0^*(1430)$	1183	1183	1183
$m_{\Phi_I}[f_0(980)]$	976	964	917
$m_{\Phi_{II}}[f_0(1370)]$	1377	1367	1343
m_{σ_I}	980	980	980
$m_{\sigma_{II}}$	1383	1383	1383
$m_{\sigma_{III}}$	1706	1697	1675
Δ_I	4.1 10^{-3}	1.7 10^{-2}	6.9 10^{-2}
Δ_{II}	4.4 10^{-3}	1.2 10^{-2}	3.0 10^{-2}
Δ_{III}	2.3 10^{-3}	7.6 10^{-3}	2.0 10^{-2}
$\langle \bar{u}u \rangle^{\frac{1}{3}}$	-208	-208	-208
$\langle \bar{s}s \rangle^{\frac{1}{3}}$	-207	-207	-207

pure glueball is also well reproduced. Whatever χ_0, this mass difference is always positive at variance with results of [11]. When $m_{\Phi_{III}} = m_{f_0} = 1710$ MeV, the coupling is weak whatever the value of χ_0. In order to get the mass difference $(m_{\Phi_{III}} - m_{\sigma_{III}}) \approx 60$ MeV, one should consider $\chi_0 \approx 80$ MeV, yielding a much too small value for the mass of the $f_0(980)$ ($m_{f_0(980)} = 573$ MeV).

Another possibility would consist in giving the contents of each physical field $\tilde{\Phi}_j$ in the three components $\tilde{\sigma}_j$. This would amount to calculate the Euler mixing angles of the mesons. However, the definition of these angles assumes that the physical states are orthogonal to each other and consequently that the mixing is energy independent. The latter assumption is not fulfilled here. Exceptional care has then to be stressed when diagonalization is carried out together with the on-mass shell definitions of associated quantities.

After diagonalization, the right-hand side of (11) reduces to:

$$\frac{1}{2\beta\Omega} \sum_q \sum_i \tilde{\Psi}_{iq}^T \lambda_{ii}(q^2) \tilde{\Psi}_{i-q} \tag{20}$$

where

$$\tilde{\Psi}_{iq} = \left(V^{-1}(q^2)\tilde{\sigma}_q\right)_i \tag{21}$$

and where the q^2 dependent eigenvalues $\lambda_{ii}(q^2)$ are given by

$$\lambda_{ii}(q^2) = \left(V^{-1}(q^2)S_R^{-1}V(q^2)\right)_{ii}, \tag{22}$$

the matrix V being the orthogonal eigenvector matrix. Let us introduce the matrix

$$G(q^2) = \partial_{q^2}\left[diag\left(\lambda_{ii}(q^2)\right)\right] = \left[\partial_{q^2}\left(V^{-1}(q^2)\right)V(q^2), diag\left(\lambda_{ii}(q^2)\right)\right] + V^{-1}(q^2)\left(\partial_{q^2}S_R^{-1}\right)V(q^2) \tag{23}$$

whose diagonal elements calculated on shell can be simplified into:

$$g^{-2}_{\Phi_i q\bar{q}} = G_{ii}(-m^2_{\Phi_i}) = \left[V^T(-m^2_{\Phi_i})\left(\partial_{q^2}S_R^{-1}\right)V(-m^2_{\Phi_i})\right]_{ii}. \tag{24}$$

$V(-m^2_{\Phi_i})$ is the eigenvector matrix whose first column is evaluated at $-m^2_{\Phi_I}$,the second one at $-m^2_{\Phi_{II}}$ and its last one at $-m^2_{\Phi_{III}}$. Had we no meson-meson mixing, Eq. (24) would define the coupling constants of the mesons to the quarks. Note that we loose the orthogonality of the matrix V ($V^{-1} \neq V^T$). If the diagonal elements of the matrix S_R^{-1} had the form

$$(S_R^{-1})_{ii} = A_i q^2 + B_i \tag{25}$$

with A_i and B_i momentum independent as well as its nondiagonal elements, one could show that the expression (20) can be *identically* written:

$$\frac{1}{2\beta\Omega}\sum_q\sum_i \tilde{\Phi}^T_{iq}\left(q^2 + m^2_{\Phi_i}\right)\tilde{\Phi}_{i-q} \tag{26}$$

with

$$\tilde{\Phi}_{iq} = g^{-1}_{\Phi_i q\bar{q}}\left(\left[V(-m^2_{\Phi_i})\right]^{-1}\right)_{ij}\tilde{\sigma}_{qj} \equiv \sum_{j=I,II,III} d_{ij}(-m^2_{\Phi_i})\left(\partial_{q^2}S_R^{-1}\right)_{jj}\tilde{\sigma}_{jq}. \tag{27}$$

The demonstration of this result is quite tedious. We do not repeat it here. Moreover, the assumptions above are not totally valid. One must be aware of the fact that neither the equality nor the identity of (27) are valid because the

Table 3
$u\bar{u}$,$s\bar{s}$ and glue contents of the $f_0(980)$, $f_0(1300)$ and $f_0(1500)$

	$\chi_0 = 350$ MeV	$\chi_0 = 200$ MeV	$\chi_0 = 125$ MeV
$d_{f_0(980)u}$	0.995	0.981	0.924
$d_{f_0(980)s}$	-0.013	-0.041	-0.117
$d_{f_0(980)\chi}$	0.098	0.188	0.664
$d_{f_0(1370)u}$	0.046	0.136	0.337
$d_{f_0(1370)s}$	0.939	0.842	0.698
$d_{f_0(1370)\chi}$	-0.342	-0.522	-0.632
$d_{f_J(1500)u}$	0.088	0.137	0.180
$d_{f_J(1500)s}$	-0.345	-0.537	-0.707
$d_{f_J(1500)\chi}$	-0.935	-0.832	-0.684

Table 4
$u\bar{u}$,$s\bar{s}$ and glue contents of the $f_0(980)$, $f_0(1300)$ and $f_J(1710)$

	$\chi_0 = 350$ MeV	$\chi_0 = 200$ MeV	$\chi_0 = 125$ MeV
$d_{f_0(980)u}$	0.998	0.993	0.979
$d_{f_0(980)s}$	-0.008	-0.025	-0.066
$d_{f_0(980)\chi}$	0.063	0.113	0.192
$d_{f_0(1370)u}$	0.016	0.048	0.123
$d_{f_0(1370)s}$	0.993	0.978	0.945
$d_{f_0(1370)\chi}$	-0.121	-0.205	-0.304
$d_{f_J(1710)u}$	0.061	0.105	0.162
$d_{f_J(1710)s}$	-0.122	-0.209	-0.321
$d_{f_J(1710)\chi}$	-0.991	-0.972	-0.933

quantity $\partial_{q^2} S_R^{-1}$ is now q^2 dependent. However, since the q^2 dependence of the coefficients A_i and B_i is small in the investigated region, we still consider that Eq. (27) defines the physical fields of the scalars $f_0(980)$ and $f_0(1370)$ and of the glueball. We choose to evaluate the quantity $\partial_{q^2}(S_R^{-1})_{jj}$ at the respective zeroes $(-m_{\sigma_i}^2)$ of the diagonal elements of S_R^{-1}. With our prescription, the quantity $d_{12}(-m_{\Phi_1}^2)$, for instance, provides the contents in $s\bar{s}$ excitations of the $f_0(980)$. We then denote it $d_{f_0(980)s}$ in tables (3) and (4). These tables confirm the results of tables (1) and (2) in the sense that the physical mesons

are nearly pure $u\bar{u}$, $s\bar{s}$ or gluonic excitations for large χ_0 while significant mixing appears for smaller χ_0. Note also that the mixing is larger for the $f_0(1500)$ than for the $f_0(1710)$ due to the fact that larger the mass of the glueball, larger its decoupling. According to us, the numbers given here are however less transparent than the Δ_i to quantify the glue contents.

5 Conclusion

The SU(3) scaled effective model developped in Ref. [13] provides some glue contents to two mesons of the scalar nonet as well as some $q\bar{q}$ contents to the glueball. Since the model contains two free parameters, one of them (here, the vacuum constituent up quark mass) can always be choosen to reproduce one of the scalars (we chose $a_0(980)$). The other one (the gluon condensate χ_0) can remain free. As briefly reviewed in the introduction, the identification of the scalar nonet as well as the one of the glueball candidate is not clear. Here we have choosen the values of Ref. [14] for the nonet while we have considered two possible candidates for the glueball: the $f_0(1500)$ [8,9] and the $f_J(1710)$ [10]. The best result for the nonet is obtained with a large gluon condensate ($\chi_0 \approx 350$ MeV). Note however that the mass of the $K_0^*(1430)$ is always too small. In that case, the $f_0(1500)$ or the $f_J(1710)$ can be said to be pure glueballs, their $q\bar{q}$ contents increasing their mass of 18 MeV and 4 MeV respectively. If one wants to reproduce the contribution of ≈ 60 MeV of the $q\bar{q}$ excitations to the glueball [10], one has to use $\chi_0 \approx 200$ MeV for $f_0(1500)$ and $\chi_0 \approx 80$ MeV for $f_J(1710)$. In the former case, the nonet is still not badly reproduced but in the latter one, the agreement is completely destroyed. In conclusion, the condition for which our model reproduces the scalar nonet (except the $K_0^*(1430)$) together with the glueball is that only a tiny mixing exists between them.

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